Harmonic crossover exponents in O(n) models with the pseudo-ϵ expansion approach

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Abstract

We determine the crossover exponents associated with the traceless tensorial quadratic field, the third- and fourth-harmonic operators for O(n) vector models by re-analyzing the existing six-loop fixed dimension series with pseudo-ϵ expansion. Within this approach we obtain the most accurate theoretical estimates that are in optimum agreement with other theoretical and experimental results.

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I. INTRODUCTION

For many years the critical behavior of O(n) vector models has caught a lot of attentions since most of physical systems undergoing second-order phase transitions belong to the O(n) universality classes (see Ref. [1] for a recent review). Thus a precise determination of universal quantities as critical exponents, amplitude ratios, etc. has became necessary. The critical behavior of these physical systems may be obtained by field theoretical investigations essentially based on the Landau-Ginzburg-Wilson Hamiltonian

$$\mathcal{H} = \int d^d x \left[\frac{1}{2} \partial_\mu \vec{\phi} \cdot \partial_\mu \vec{\phi} + \frac{1}{2} r \vec{\phi} \cdot \vec{\phi} + \frac{1}{4!} u (\vec{\phi} \cdot \vec{\phi})^2 \right], \tag{1}$$

where $\vec{\phi}(x)$ is an n-component real field. An interesting issue in these systems is to determine the behavior of the Hamiltonian (1) under the presence of perturbation terms

$$\mathcal{H} = \int d^d x \left[\frac{1}{2} \partial_\mu \vec{\phi} \cdot \partial_\mu \vec{\phi} + \frac{1}{2} r \vec{\phi} \cdot \vec{\phi} + \frac{1}{4!} u (\vec{\phi} \cdot \vec{\phi})^2 + h_p \mathcal{P} \right], \tag{2}$$

where $h_p(x)$ is an external field coupled to $\mathcal{P}(x)$. In fact, if $\mathcal{P}$ is an eigenoperator of the RG transformations, the singular part of the Gibbs free energy becomes a scaling function in the limit of reduced temperature $t \to 0$ and $h_p \to 0$, and can be written as

$$\mathcal{F}_{\rm sing}(t, h_p) \approx |t|^{d\nu} \widehat{\mathcal{F}} \left(h_p |t|^{-\phi_p} \right), \tag{3}$$

where $\phi_p \equiv y_p \nu$ is the crossover exponent associated with the perturbation $\mathcal{P}$ and y_p is the Renormalization Group (RG) dimension of $\mathcal{P}$. Moreover, one usually defines the indices β_p and γ_p which describe the low-temperature singular behavior of the average $\langle \mathcal{P}(x) \rangle \sim |t|^{\beta_p}$ and of the suscettivity $\chi_{\mathcal{P}} = \int d^d x \langle \mathcal{P}(x) \mathcal{P}(0) \rangle_c \sim t^{-\gamma_p}$. They satisfy the scaling relations

$$\beta_p = 2 - \alpha - \phi_p, \qquad \gamma_p = -2 + \alpha + 2\phi_p. \tag{4}$$

Among the perturbation operators, particularly relevant from the experimental and phenomenological point of view are the so-called harmonic ones [2–4]

$$\begin{aligned} \mathcal{P}_2(x) &= \phi_i(x)\phi_j(x) - \delta_{ij} \frac{1}{n} \vec{\phi}(x) \cdot \vec{\phi}(x), \\ \mathcal{P}_3(x) &= \phi_a \phi_b \phi_c - \frac{\vec{\phi} \cdot \vec{\phi}}{n+2} (\phi_a \delta_{bc} + \phi_b \delta_{ac} + \phi_c \delta_{ab}), \\ \mathcal{P}_4(x) &= \phi_a \phi_b \phi_c \phi_d - \frac{\vec{\phi} \cdot \vec{\phi}}{n+4} (\delta_{ab} \phi_c \phi_d + 5\text{perm.}) + \frac{(\vec{\phi} \cdot \vec{\phi})^2}{(n+4)(n+2)} (\delta_{ab} \delta_{cd} + 2\text{perm.}), \end{aligned} \tag{5}$$

called second, third, and fourth harmonic operator respectively. In the following we denote the crossover exponent of $\mathcal{P}_i$ as ϕ_i and its RG dimension as y_i. Higher order harmonic operators are generally reputed to be irrelevant at the three-dimensional O(n) fixed point [3], thus we will not consider them here.

The crossover exponent ϕ_2 associated with the traceless tensor field $\mathcal{P}_2(x)$ reveals the instability of the O(n)-symmetric theory against anisotropy [2,5–7]. It characterizes the phase diagram at the multicritical point where two critical lines O(n) and O(m) symmetric meet. In some cases this gives rise to a critical theory with enlarged O($n+m$) symmetry [8–11]. Multicritical behavior arises in several different contexts in physics: in anisotropic antiferromagnets in a uniform magnetic field [6,11], in high T_c superconductors (see e.g. Ref. [12] and note that in the SO(5) theory of superconductivity [13] the multicritical point is effectively O(5) symmetric), in colossal magnetoresistance materials [14], in certain theories of strong interactions [15], etc. This list is far from being exhaustive, we only quote some examples in very far away fields. For the XY model ($n = 2$) the traceless tensor field $\mathcal{P}_2(x)$ and its correlation function are connected with the second-harmonic order parameter in density-wave systems [16,17], which characterizes some liquid crystals at the nematic-smectic-A transition [16–23]. The structure factor of the secondary order parameter $\mathcal{P}_2$, that within RG methods has been determined in- [16,24] and out-of-equilibrium [25], has been experimentally measured using X-ray scattering techniques [22,23]. Finally the RG dimension y_2 enters in the study of crossover effects in diluted Ising antiferromagnets with n-fold degenerate ground state [26], in models with random anisotropy [27], at certain quantum phase transitions [28], and in other more complicated situations [29]. Even this list is far from being exhaustive.

The third-harmonic crossover exponent determines the phase diagrams at the smectic-A hexatic-B point in liquid crystals [18,23], in materials exhibiting structural normal-incommensurate phase transitions [30–32], and at the trigonal-to-pseudotetragonal transition [33]. For $n = 0$, ϕ_3 is related to the partition function exponent of nonuniform star polymers with three arms [34]. Finally, for $n \geq 2$ it determines the stability of O(n) fixed points against $n+1$-state Potts-like perturbations [35] as it happens in the presence of stress or particular magnetic fields [33,36].

The fourth-harmonic exponent ϕ_4 is mainly related to the stability of the O(n) fixed point against fourth-order anisotropy [8], as e.g. the cubic one [7]. It is worth mentioning that for $n = 1$, even if the operators $\mathcal{P}_i(x)$ can not be defined through Eqs. (5), all the ϕ_i have non trivial values. This fact has an interpretation in terms of a gas of n-colors loops (see e.g. [37]) in the limit $n \to 1$.

The exponents ϕ_i and y_i with $i = 2, 3, 4$ have been analyzed in the past with different theoretical methods, in the framework of the ϵ-expansion [38–41,2,3,18,8,34], from the analysis of high-temperature expansion [9], by means of Monte Carlo simulations [42–44], in the large n approach [3,45,46], and in the fixed-dimension perturbative expansion [18,24,34].

The aim of this paper is to determine the crossover exponents ϕ_i and the RG dimensions y_i by re-analyzing the three-dimensional six-loop perturbative series [24,34] with the pseudo-ϵ expansion trick [47], since in many cases this method provided the most accurate results in the determination of critical quantities (see, e.g., Refs. [48–52]). The idea behind this trick is very simple: one has to multiply the linear term of the β function by a parameter τ, find the fixed points (i.e. the zeros of the β function) as series in τ and analyze the results as in the ϵ expansion. The critical exponents are obtained as series in τ inserting the fixed-point expansion in the appropriate RG functions. With this trick the cumulation of the errors coming from the non-exact knowledge of the fixed point and from the uncertainty in the resummation of the exponents is avoided, obtaining very precise results even without exploiting advanced resummation techniques as the conformal mapping one [48]. Note that now, differently from ϵ expansion, only the value at $\tau = 1$ makes sense, since the original series are obtained in fixed dimension $d = 3$.

The paper is organized as follows. In Sec. II we analyze the quadratic crossover exponents, in Sec. III the cubic and quartic ones. In Sec. IV we report all the pseudo-ϵ estimates for harmonic exponents and compare them with other theoretical and experimental ones.

II. QUADRATIC CROSSOVER EXPONENTS

The six-loop RG perturbative series in the three-dimensional approach for the second harmonic operators were computed in Ref. [24], whereas the β function (necessary to find the stable fixed point) is reported for general n in Ref. [53]. By using these series, one obtains the pseudo-ϵ expansion of all the second-harmonic exponents.

We first consider the crossover exponent ϕ_2. As a typical example, the perturbative expression in the parameter τ for $n = 2$ reads

$$\phi_2^{n=2} = 1 + \frac{\tau}{10} + \frac{317\tau^2}{6750} + 0.01688\tau^3 + 0.01005\tau^4 + 0.00227\tau^5 + 0.00457\tau^6 + O(\tau^7)\,. \quad (6)$$

At least up to the presented number of loops the series does not behave as asymptotic with factorial growth of coefficients. Although the series has not alternating signs, which is a key point to ensure some kind of convergence, one can try to apply a simple Padé summation. The results for $n = 2$ are displayed in Table I. All the approximants possess poles on the real positive axis. Some of them are close to $\tau = 1$ and the estimate of ϕ_2 on their basis should be considered unreliable. Anyway some of these approximants have poles "far" from $\tau = 1$, where the series must be evaluated. Thus one may expect the presence of such poles not to influence the approximant at $\tau = 1$. Indeed all such Padé results are very close since lower orders. Hereafter we choose as final estimate the average of those six-loop order Padé without poles in $0 \le \tau \le 2$, and as error bar we take the maximum deviation of the final estimate from the four-, five and six-loop Padé. The five-loop estimates are analogously obtained, considering the maximum deviations up to three loops. Within this procedure we obtain $\phi_2^{n=2} = 1.178(15)$ at five-loop and 1.181(7) at six-loop. Although in good agreement

TABLE I. Padé table for $\phi_2^{n=2}$ in pseudo-ϵ expansion. The two integer numbers N and M denote the corresponding $[N/M]$ Padé. The location of the positive real pole closest to the origin is reported in brackets.

	$N=0$	$N=1$	$N=2$	$N=3$	$N=4$	$N=5$	$N=6$
$M=0$	1	1.1	1.14696	1.16385	1.17390	1.17616	1.18073
$M=1$	$\underline{1.11111}_{[10]}$	$\underline{1.18855}_{[2.1]}$	$\underline{1.17332}_{[2.8]}$	$\underline{1.18868}_{[1.7]}$	$\underline{1.17683}_{[4.4]}$	$\underline{1.17166}_{[0.5]}$	
$M=2$	$\underline{1.15870}_{[4.0]}$	$\underline{1.17369}_{[2.7]}$	$\underline{1.17925}_{[2.3]}$	$\underline{1.17952}_{[2.3]}$	$\underline{1.18113}_{[2.1]}$		
$M=3$	$\underline{1.17021}_{[3.2]}$	$\underline{1.19549}_{[1.4]}$	$\underline{1.17952}_{[2.3]}$	$\underline{1.17920}_{[0.1]}$			
$M=4$	$\underline{1.17818}_{[2.7]}$	$\underline{1.17773}_{[2.7]}$	$\underline{1.18127}_{[2.03]}$				
$M=5$	$\underline{1.17771}_{[2.7]}$	$\underline{1.17814}_{[2.7]}$					
$M=6$	$\underline{1.18271}_{[2.2]}$						

TABLE II. Padé table for $y_2^{n=2}$ in pseudo-ϵ expansion. The two integer numbers N and M denote the corresponding $[N/M]$ Padé. The location of the positive real pole closest to the origin is reported in brackets. The final estimate is $y_2^{n=2} = 1.763(4)$.

	$N=0$	$N=1$	$N=2$	$N=3$	$N=4$	$N=5$	$N=6$
$M=0$	2	1.8	1.76089	1.76705	1.76215	1.76707	1.76041
$M=1$	1.81818	$\underline{1.75138}_{[5.1]}$	1.76621	1.76432	1.76461	1.76424	
$M=2$	1.77061	1.76756	$\underline{1.76376}_{[9.8]}$	1.76459	1.76443		
$M=3$	1.76774	$\underline{1.77550}_{[0.6]}$	1.76471	$\underline{1.76433}_{[13]}$			
$M=4$	1.76322	1.76505	$\underline{1.76405}_{[3.4]}$				
$M=5$	$\underline{1.76630}_{[4.4]}$	1.76444					
$M=6$	1.76162						

with other theoretical estimates, we do not retain safe such estimates, since the presence of so many poles in the Padé can cause systematic deviations from the actual value. This can be traced back to the fact that the series has not alternating signs. To improve the estimates, one can try to resum the series by means of the Padé-Borel-Leroy (PBL) method (see e.g. [48]) or more advanced ones, but due to the monotonic character of the signs of the series this leads the majority of the approximants to be defective. The resulting few good approximants do not allow a safe determination of the quantities analyzed. The same scenario is found for all other values of n.

To achieve a reliable estimate of ϕ_2, one has to consider series that have alternating signs. This can be done by considering the RG dimension $y_2 = \phi_2/\nu$. The pseudo-ϵ expansion of y_2 for general n is

$$
\begin{aligned}
y_2 = {} & 2 - \frac{2}{(n+8)}\tau + \frac{2(-392-78n+5n^2)}{27(8+n)^3}\tau^2 \\
& + \frac{362.271 + 65.4612\,n + 16.8274\,n^2 + 6.59007\,n^3 + 0.192062\,n^4}{(8+n)^5}\tau^3 \\
& - \frac{19417.6+10881.8\,n+2401.97\,n^2-29.3771\,n^3-83.6197\,n^4-5.54754\,n^5-0.0986411\,n^6}{(n+8)^7}\tau^4 \\
& + \frac{1.66461\,10^6+981069.\,n+215076.\,n^2+30474.1\,n^3+8065.54\,n^4+1734.33\,n^5+132.809\,n^6+4.22169\,n^7+0.0573847\,n^8}{(8+n)^9}\tau^5 \\
& - \Big[\frac{1.54538\,10^8+1.25687\,10^8 n+4.50851\,10^7 n^2+9.02640\,10^6 n^3+7.98957\,10^5 n^4-84675.5n^5-31129.6n^6-2974.43n^7}{(8+n)^{11}} \\
& \frac{-133.244n^8-3.26010n^9-0.0369419n^{10}}{(8+n)^{11}}\Big]\tau^6\,,
\end{aligned}
\qquad (7)
$$

that has alternating signs for $n \lesssim 6$. In fact, we get a small number of Padé with poles on the real positive axis, as one may appreciate from Table II where the results for $n=2$ are displayed. The goodness of the Padé persists increasing n up to $n \simeq 6$ and then the results get worse. All the final data are shown in Table IV, where we also report the result for $n=16$, that has to be taken with care since in this case the series is not alternating in signs.

We resum the perturbative series by the PBL method too. The number of defective approximants is very low and one may obtain a different estimate of the quantity y_2. E.g. for $n = 2$ the result are incredibly stable, suggesting $y_2 = 1.7645(3)$ which is compatible with the Padé values, but it has a smaller error. This great stability within the PBL resummation makes us decide to report as final estimates the simple Padé ones, in order to avoid an underestimation of the uncertainties.

Exploiting the scaling relations (4), we can apply the previous procedure to characterize the critical exponents β_2 and γ_2. Unfortunately these series have no alternating signs for all values of n, resulting in a bad determination of their actual value. In Table IV we display ϕ_2, β_2, and γ_2 as obtained by using scaling relations from y_2 and the most accurate estimates of standard critical exponents in the O(n) universality class [54].

III. THIRD- AND FOURTH-HARMONIC CROSSOVER EXPONENTS

In this section we consider the critical exponents of the third- and fourth-harmonic operators.

The six-loop three dimensional series relevant for $\mathcal{P}_3$ were calculated in Ref. [34]. Even in this case only the direct estimate of y_3 gives rise to a reliable result, since the series for β_3, γ_3, and ϕ_3 have not alternating signs. The pseudo-ϵ expansion of y_3 for general n is

$$\begin{aligned} y_3 = {} & \frac{3}{2} - \frac{6}{n+8}\tau - \frac{144+44n-14n^2}{9(n+8)^3}\tau^2 \\ & + \frac{404.981 + 281.073\,n + 78.7256\,n^2 + 20.9419\,n^3 + 0.613212\,n^4}{(n+8)^5}\tau^3 \\ & + \frac{-22461.1 - 17805.3\,n - 3307.73\,n^2 + 417.392\,n^3 + 244.052\,n^4 + 15.6783\,n^5 + 0.289476\,n^6}{(n+8)^7}\tau^4 \\ & + \frac{1.67818\,10^6 + 1.44153\,10^6\,n + 420045.\,n^2 + 92283.5\,n^3 + 24863.3\,n^4 + 4900.83\,n^5 + 366.716\,n^6 + 11.7112\,n^7 + 0.162327\,n^8}{(n+8)^9}\tau^5 \\ & - \left[\frac{1.63843\,10^8 + 1.78385\,10^8 n + 7.35887\,10^7 n^2 + 1.53121\,10^7 n^3 + 968499.n^4 - 318541.n^5 - 87011.6n^6 - 8034.60n^7}{(n+8)^{11}}\right. \\ & \left. \frac{-360.085n^8 - 8.93464n^9 - 0.102502n^{10}}{(n+8)^{11}}\right]\tau^6\,, \end{aligned} \tag{8}$$

TABLE III. Padé table for $y_3^{n=0}$ in pseudo-ϵ expansion. The location of the positive real pole closest to the origin is reported in brackets.

	$N=0$	$N=1$	$N=2$	$N=3$	$N=4$	$N=5$	$N=6$
$M=0$	1.5	0.75	0.71875	0.73111	0.72040	0.73290	0.71383
$M=1$	1	$0.71739_{[24]}$	0.72761	0.72537	0.72617	0.72535	
$M=2$	0.84706	0.73152	$0.72529_{[47]}$	0.72603	0.72573		
$M=3$	0.78599	$0.71920_{[3.5]}$	0.72621	$0.72572_{[60]}$			
$M=4$	0.75533	0.73227	$0.72530_{[6.5]}$				
$M=5$	0.74240	$0.68973_{[1.3]}$					
$M=6$	0.73215						

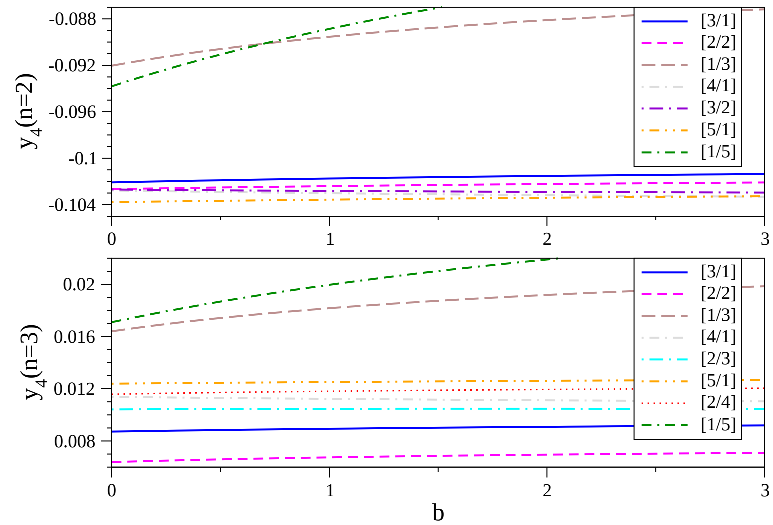

FIG. 1. Non-defective four-, five, and six-loop PBL approximants for the RG dimension y_4 for $n = 2, 3$.

which has alternating signs for $n \lesssim 5$. To show the goodness of the Padé summation we report in Table III the results for $n = 0$. The final estimate from this table is $y_3 = 0.725(29)$ [$y_3 = 0.731(35)$] at six-loop [five-loop], where to get the estimate and the error bar we used the procedure outlined in the previous section. Similar good Padé tables are found for higher values of n, up to $n \simeq 5$. All the final results are reported in Table IV. We also check our results using PBL resummation, but again the uncertainty we get in such manner is too small to be considered safe.

It is worth noting that for the partition function exponent p of non-uniform star polymers with three arms [34], we obtain the pseudo-ϵ series

$$p = 3(\gamma + \nu)/2 + \phi_3 = 3 + \frac{\tau^2}{16} - 0.009278\,\tau^3 + 0.005889\,\tau^4 - 0.014350\,\tau^5 + 0.015458\,\tau^6\,, \quad (9)$$

which (by means of simple Padé) leads to $p = 3.055(11)$. This value compares well with other estimates [34] and with the one obtained from y_3 and the most accurate theoretical estimates of γ and ν [54] leading to $p = 3.043(18)$.

Finally, let us consider the fourth-harmonic exponent. It has been shown in Refs. [55,8] that the RG dimension y_4 is related to the exponent characterizing the stability of the O(n) fixed point against cubic anisotropy. Thus we can use the six-loop series of the cubic model [56] to obtain the pseudo-ϵ expansion

$$\begin{aligned} y_4 = {} & 1 - \frac{12}{n+8}\tau + \frac{4(680 + 62\,n + 23\,n^2)}{27(8+n)^3}\tau^2 \\ & + \frac{-5200.56 - 777.127\,n + 33.2649\,n^2 + 35.7448\,n^3 + 1.25111\,n^4}{(n+8)^5}\tau^3 \\ & + \frac{328835. + 90677.3\,n + 10871.8\,n^2 + 1595.82\,n^3 + 446.805\,n^4 + 28.9975\,n^5 + 0.574653\,n^6}{(n+8)^7}\tau^4 \\ & + \frac{-3.06135\,10^7 - 1.35901\,10^7\,n - 2.55849\,10^6\,n^2 - 179042.\,n^3 + 24766.4\,n^4 + 8290.96\,n^5 + 672.157\,n^6 + 22.2622\,n^7 + 0.318104\,n^8}{(n+8)^9}\tau^5 \end{aligned}$$

TABLE IV. Final estimates from six-loop pseudo-ϵ expansion. The values of y_i are calculated directly from the series reported in the text. Whereas ϕ_i, γ_i, and β_i are obtained by means of scaling laws, using the most accurate theoretical estimates for standard critical exponents [54].

	$n=0$	$n=1$	$n=2$	$n=3$	$n=4$	$n=5$	$n=16$
y_2	-	1.733(6)	1.763(4)	1.789(3)	1.811(2)	1.830(2)	1.927(4)
ϕ_2	1	1.092(4)	1.184(3)	1.272(2)	1.356(4)	1.398(4)	1.755(4)
β_2	-	0.798(4)	0.831(3)	0.861(2)	0.891(3)	0.894(3)	0.978(4)
γ_2	-	0.294(8)	0.353(5)	0.411(4)	0.466(3)	0.504(3)	0.778(7)
y_3	0.725(30)	0.814(10)	0.891(9)	0.957(4)	1.014(4)	1.063(7)	1.31(1)
ϕ_3	0.426(18)	0.513(6)	0.598(6)	0.681(3)	0.759(4)	0.812(6)	1.19(1)
β_3	1.336(18)	1.377(6)	1.416(6)	1.453(3)	1.488(5)	1.480(7)	1.54(1)
γ_3	-0.91(4)	-0.865(13)	-0.818(12)	-0.772(6)	-0.728(6)	-0.668(11)	-0.35(2)
y_4 safe	-0.380(18)	-0.23(1)	-0.098(6)	0.012(6)	0.104(8)	0.188(8)	0.62(1)
y_4 best	-0.393(5)	-0.236(3)	-0.103(1)	0.0094(30)	0.107(5)		

$$+\left[\frac{3.18214\,10^9+1.83869\,10^9\,n+4.52163\,10^8\,n^2+5.94633\,10^7\,n^3+5.79335\,10^6\,n^4+913345.\,n^5+161861.\,n^6+14567.2\,n^7}{(n+8)^{11}}\right.$$
$$\left.\frac{+669.512\,n^8+17.0980\,n^9+0.199456\,n^{10}}{(n+8)^{11}}\right]\tau^6. \tag{10}$$

From a general analysis [1,8], it is known that y_4 is positive for $n > N_c$ and negative in the opposite case, with N_c a bit smaller than 3 [1]. The series (10) is alternating in signs for $n \lesssim 5$, but in this case the coefficients are not so small for a simple Padé summation to be effective. Thus, we apply a PBL resummation taking into account all the results coming from four-, five- and six-loop perturbative series. Fig. 1 sketches the non-defective PBL approximants for $n = 2, 3$. It is evident that several approximants are very close each other, whereas the $[1/M]$ ones are a bit far. Usually, when facing with a similar situation, the PBL approximants that are far from the mean-value are discarded in the average procedure. However, to be sure not to underestimate the error we report in Table IV both the estimates: one is the average over all PBL (safe), the other (best) is obtained discarding the $[1/M]$ approximants. For $n = 5, 16$ we report only the "safe" estimate, since the trend of higher values of the $[1/M]$ approximants seems absent.

IV. CONCLUSIONS

In this paper we determined the critical exponents associated with harmonic operators of degree 2, 3, and 4 in O(n) models by means of pseudo-ϵ expansion. All our results for y_i, ϕ_i, γ_i, and β_i are reported for $n = 0$, 1, 2, 3, 4, 5, and 16 in Table IV.

In order to make a comparison with the values in the literature we report in Table V all the most accurate theoretical estimates for y_i, as obtained by means of scaling relations (4) using the most precise determinations of standard critical exponents [54]. For all values of n our estimates are in perfect agreement with all known results and in the majority of the cases they are the most precise ones. We stress that such high accurateness should not be due to underestimation of the uncertainty, since we check our final error bars with other resummation techniques, such as Padé-Borel-Leroy and conformal mapping. However the large n results need to be discussed, since our estimates for $n \gtrsim 6$ can be affected by

TABLE V. Theoretical estimates of the RG dimension y_i as obtained by various approaches for several n: five-loop ϵ expansion (ϵ exp), six-loop fixed-dimension expansion (FD), high-temperature expansion (HT exp), Monte Carlo simulations (MC), and $1/n$ expansion at order $O(1/n^2)$ or $O(1/n)$. The results are obtained by using scaling relations, using the most precise theoretical estimates for standard critical exponents [54].

y_2		$n=1$	$n=2$	$n=3$	$n=4$	$n=5$	$n=16$
6-loop (pseudo-ϵ)		1.733(6)	1.763(4)	1.789(3)	1.811(2)	1.830(2)	1.927(4)
6-loop (FD) [24]			1.763(18)	1.787(30)	1.80(5)	1.83(5)	1.92(6)
5-loop (ϵ-exp) [8]			1.766(6)	1.790(3)	1.813(6)	1.832(8)	
MC [42]			1.755(3)	1.787(3)	1.812(2)		
MC [44]						1.815(39)	
HT exp. [9]			1.750(22)	1.758(21)			
O($1/n$) [57]				1.640	1.730	1.784	1.932
O($1/n^2$) [46,58]				1.78(14)	1.83(6)	1.88(2)	1.95(3)
y_3	$n=0$	$n=1$	$n=2$	$n=3$	$n=4$	$n=5$	$n=16$
6-loop (pseudo-ϵ)	0.725(30)	0.814(10)	0.891(9)	0.957(4)	1.014(4)	1.063(7)	1.31(1)
6-loop (FD) [34]	0.758(19)		0.895(15)	0.953(23)	1.015(31)	1.065(19)	1.310(13)
5-loop (ϵ-exp) [34]	0.739(9)		0.892(22)	0.958(42)	1.020(45)	1.064(25)	1.28(10)
O($1/n$) [57]					0.791	0.933	1.323
y_4	$n=0$	$n=1$	$n=2$	$n=3$	$n=4$	$n=5$	$n=16$
6-loop (pseudo-ϵ) best	-0.393(5)	-0.236(3)	-0.103(1)	0.0094(30)	0.107(5)		
6-loop (pseudo-ϵ) safe	-0.380(18)	-0.23(1)	-0.098(6)	0.012(6)	0.104(8)	0.188(8)	0.62(1)
6-loop (FD) [56,55]			-0.103(8)	0.013(6)	0.111(4)	0.189(10)	
5-loop (ϵ-exp) [56,8]			-0.114(4)	0.003(4)	0.105(6)	0.198(11)	
MC Ref. [43]			-0.17(2)	-0.0007(29)	0.130(24)		
O($1/n$) [57]						-0.08	0.662

systematic errors, because the perturbative series we summed have not alternating signs. Anyway, for $n = 16$ all the theoretical estimates are in good agreement, signaling that the evaluation of our uncertainty is probably good even in this case.

Let us finally compare our values with some experiments. We mention the result $\phi_2^{n=2} = 1.17(2)$ for the $(2 \to 1+1)$ bicritical point in $GdAlO_3$ [59], and $\phi_2^{n=3} = 1.279(31)$ in the $(3 \to 2+1)$ study of MnF_2 [60]. Other experimental measures of ϕ_2 can be found in Ref. [61]. The experimental results obtained for a nematic-smectic-A transition reported in Ref. [22] are $\beta_2^{n=2} = 0.76(4)$ and $\gamma_2^{n=2} = 0.41(9)$. For the third harmonic exponent we quote $\beta_3^{n=2} \simeq 1.66$ in liquid crystals [23], $\beta_3^{n=2} = 1.50(4)$ (Ref. [32]) $\beta_3^{n=2} = 1.80(5)$ (Ref. [30]) in Rb_2ZnCl_4. All these values compare well (within their own uncertainties) with our results.

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[58] In Ref. [46] were reported the four different estimates for ϕ_2, β_2, γ_2, and $\beta_2 + \gamma_2$ as obtained by Padé-Borel analysis of the corresponding $O(1/n^2)$ expansions. These four estimates obviously leads to different estimates of y_2. The value in the Tab. V is the average of the four y_2 as obtained by means of scaling relations (for the exponent ν we take the value in [54]) and the uncertainty is the maximum deviation. Obviously, the quoted error bars are only indicative.

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